Time Integration

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Time Integration

Main objective is to find unknown displacements by numerically integrating the equations of motion which is a second-order linear/nonlinear ODE

For a single degree of freedom with no damping:

$$m\ddot{x} + kx = f_{ext}(t)$$

Linear

$$m\ddot{x} + f_{\text{int}}(t) = f_{ext}(t)$$

Non-Linear

Numerical Solutions

Direct and Indirect Integration Techniques

Direct

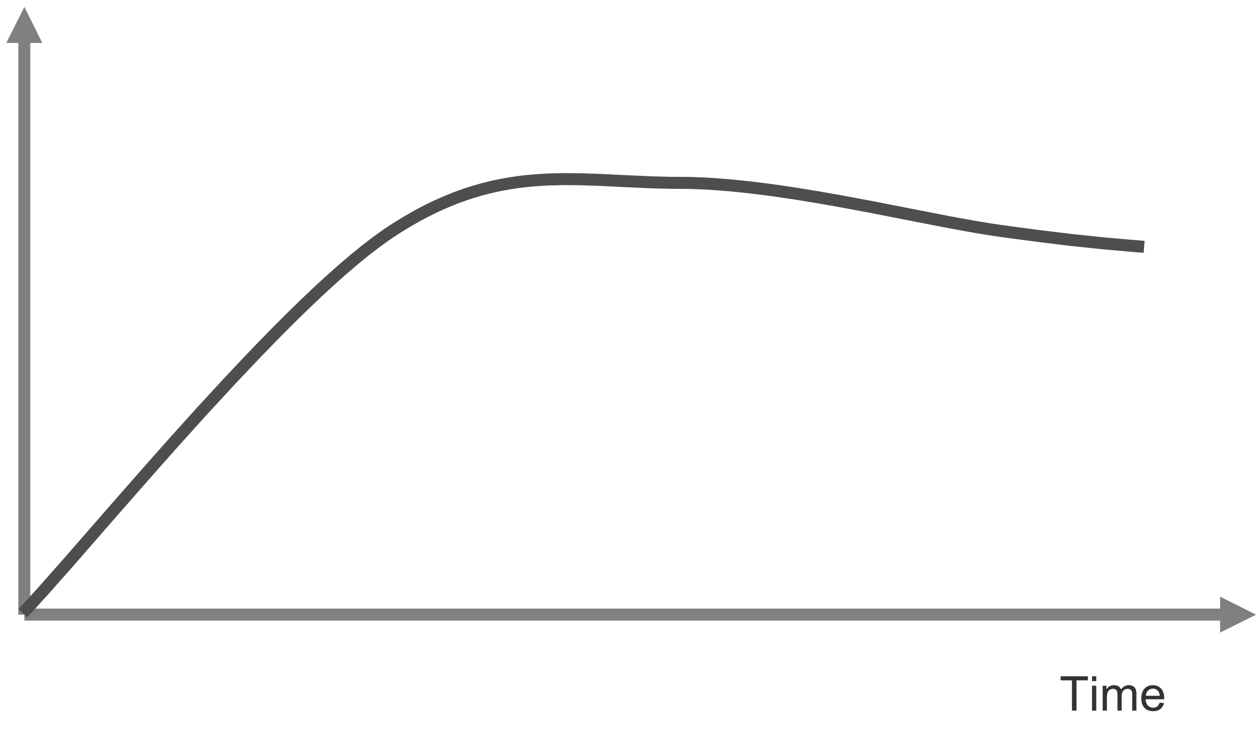
- No transformations
- Examples
  - Explicit
  - Implicit

Indirect

- Transformation
- Examples
  - Mode Superposition

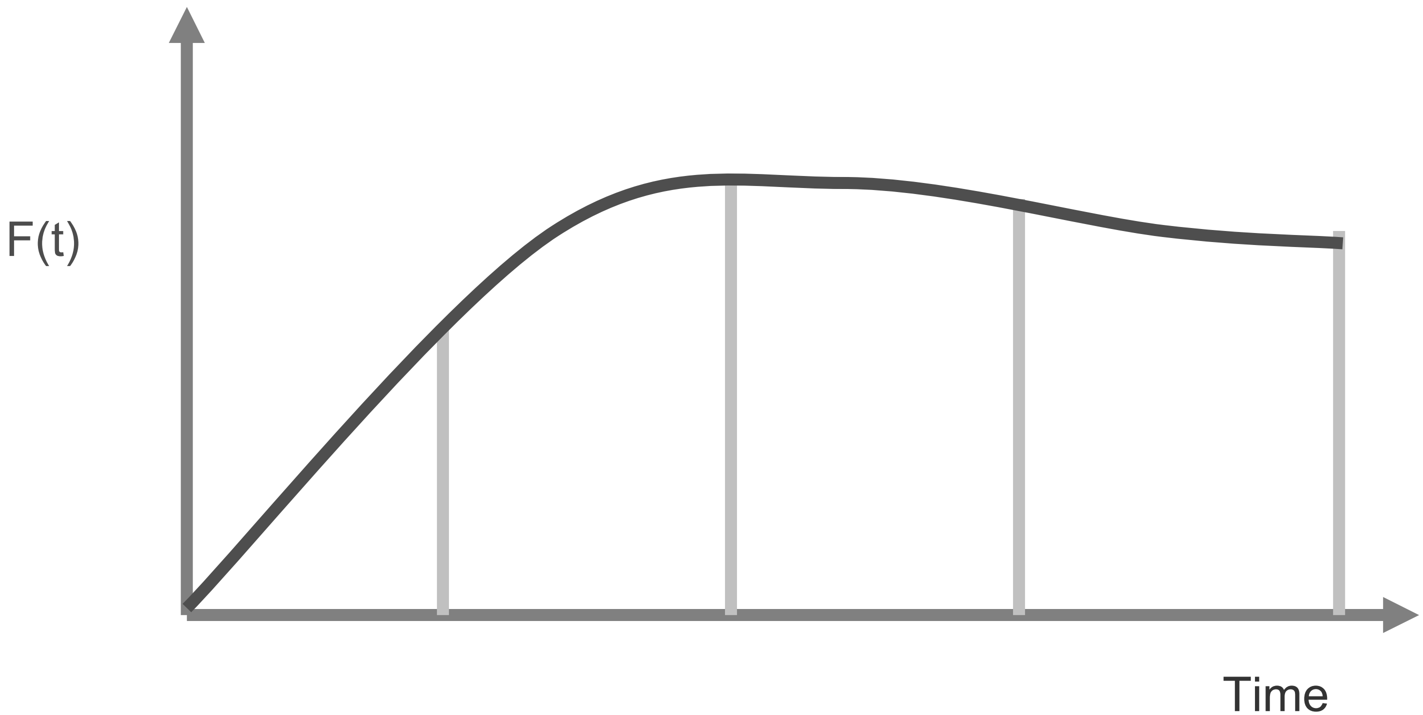
Numerical Solution - Explicit

Among many explicit methods, the central-difference technique is the most popular and is used in LS-DYNA



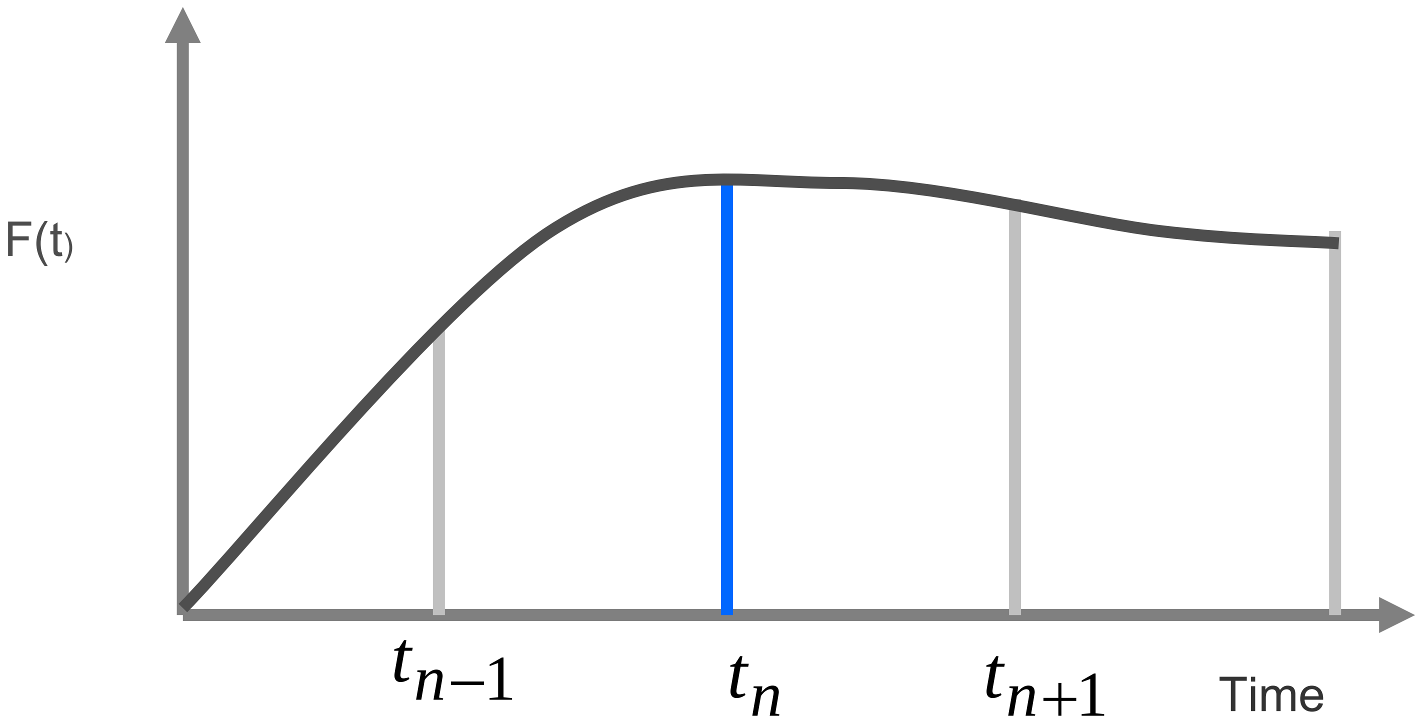
Numerical Solution - Explicit

Discretization in Time



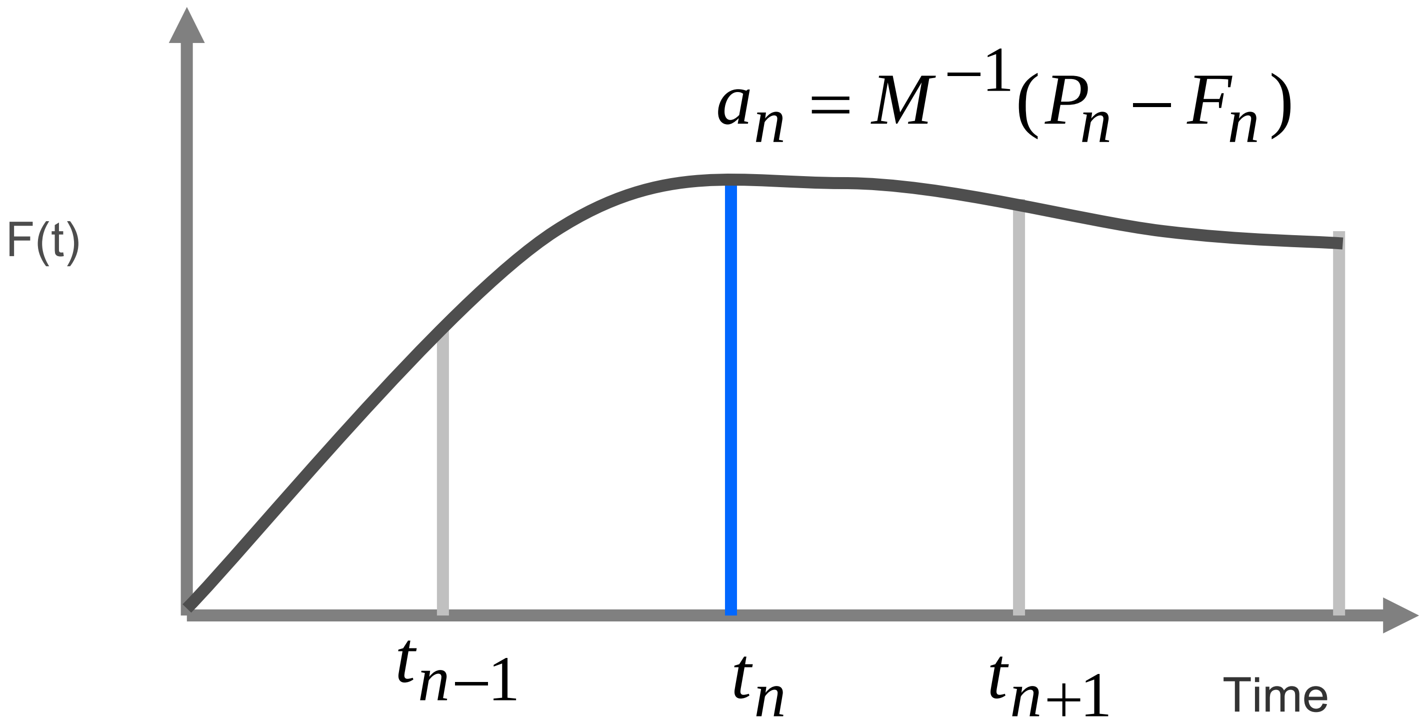
Numerical Solution - Explicit

Current Time



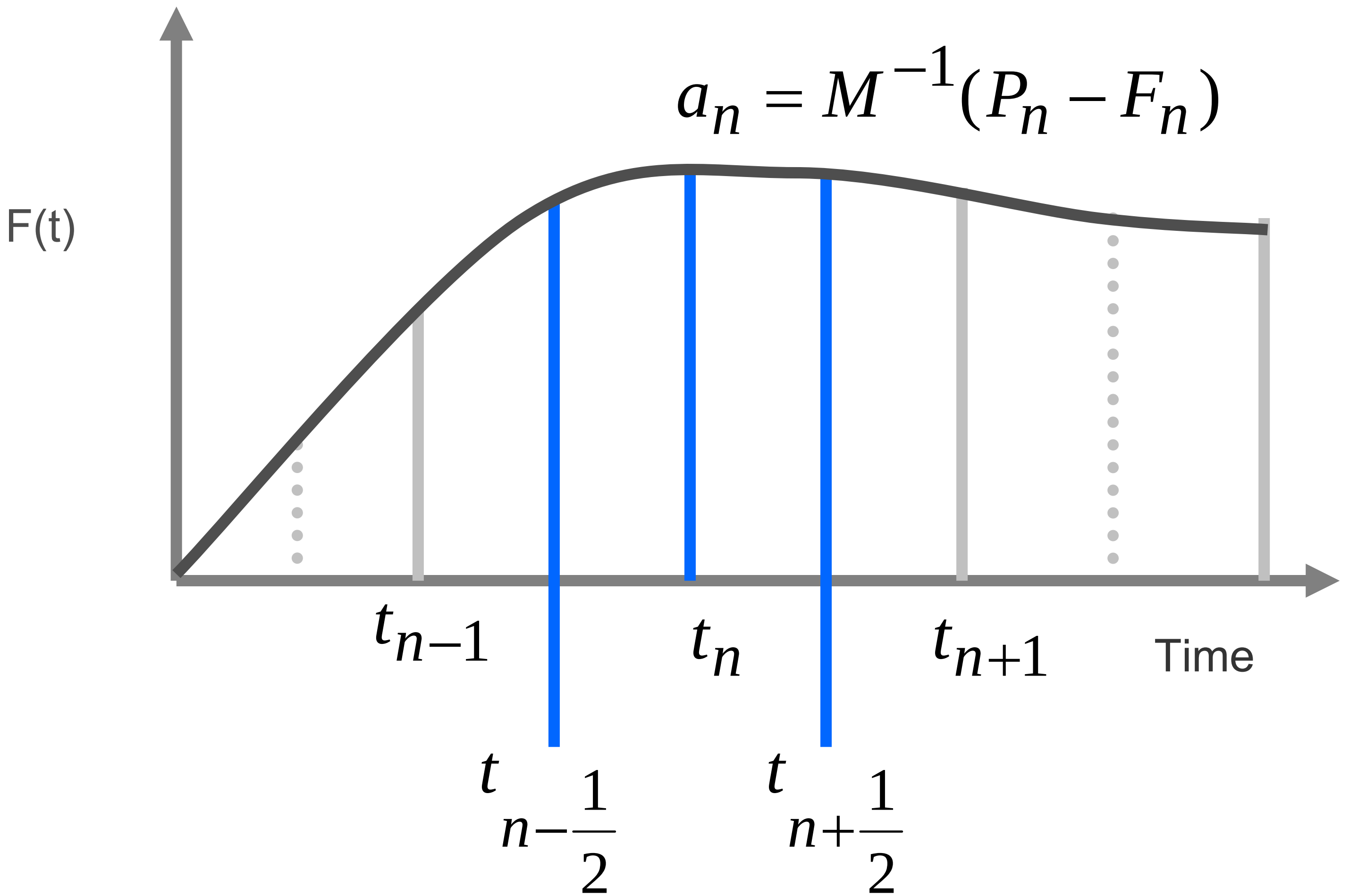
Numerical Solution - Explicit

Current Acceleration



Numerical Solution - Explicit

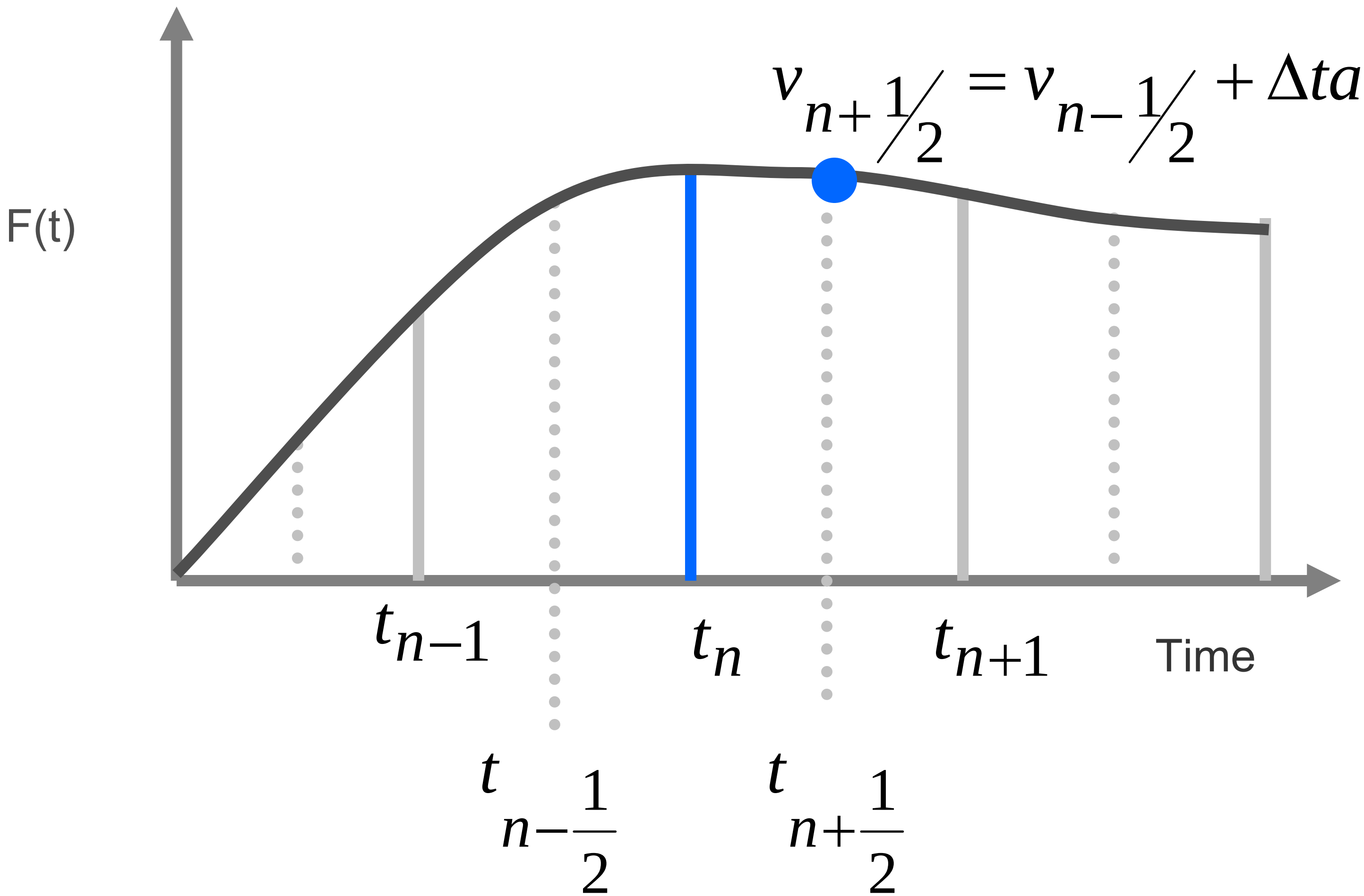
Mid-step parameteres



Numerical Solution - Explicit

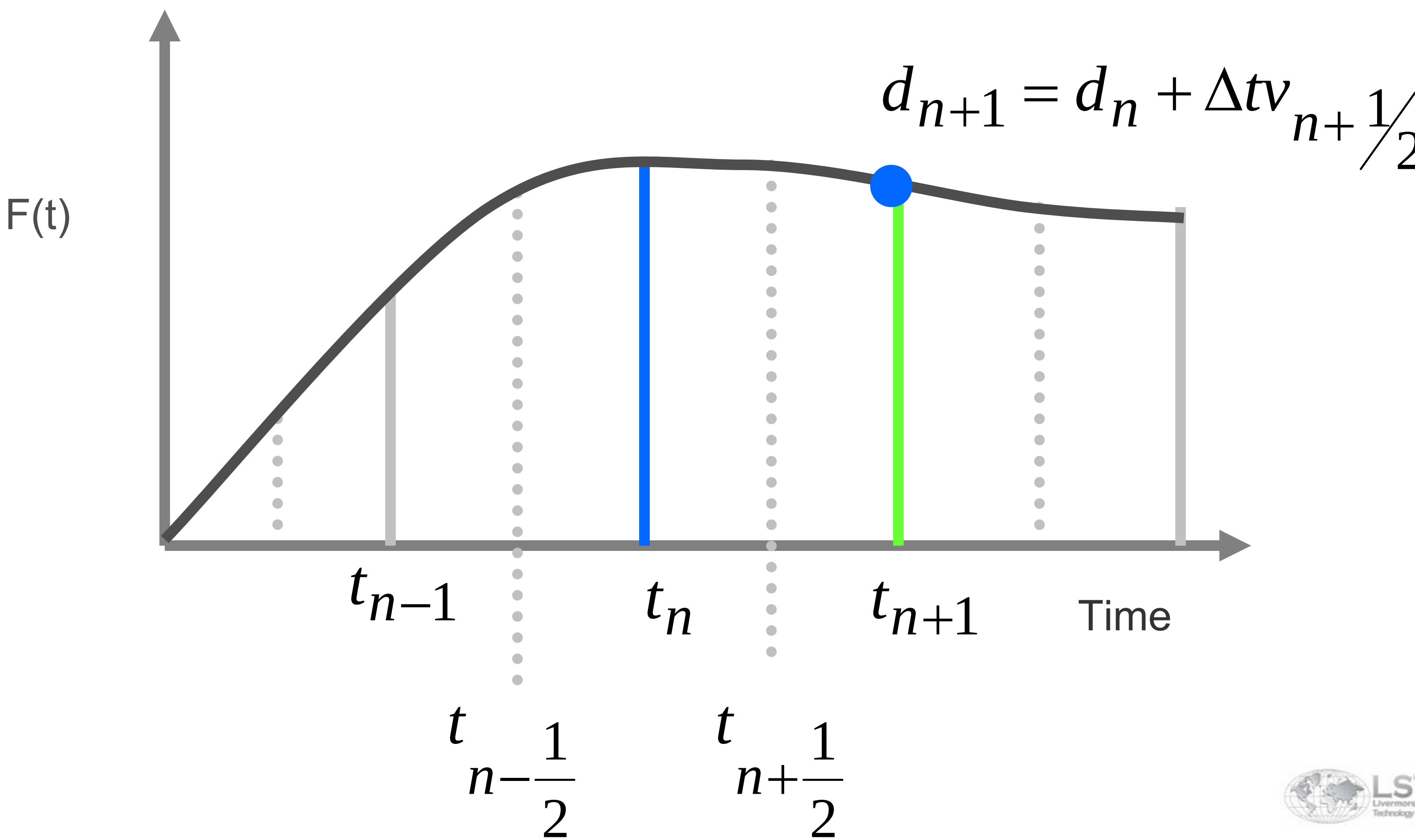
Mid-Step Velocity

$$v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + \Delta t a_n$$



Numerical Solution - Explicit

Unknown displacement



Explicit – Timestep

Choosing an incremental timestep is based on the highest natural frequency of the system

$$\Delta t < \frac{2}{\omega_{\max}}$$

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$$\Delta t < \frac{l}{c}$$

Courant-Frederick-Levy (CFL) Criteria

Characteristic Length, l_c

Element based

Computed Every Cycle

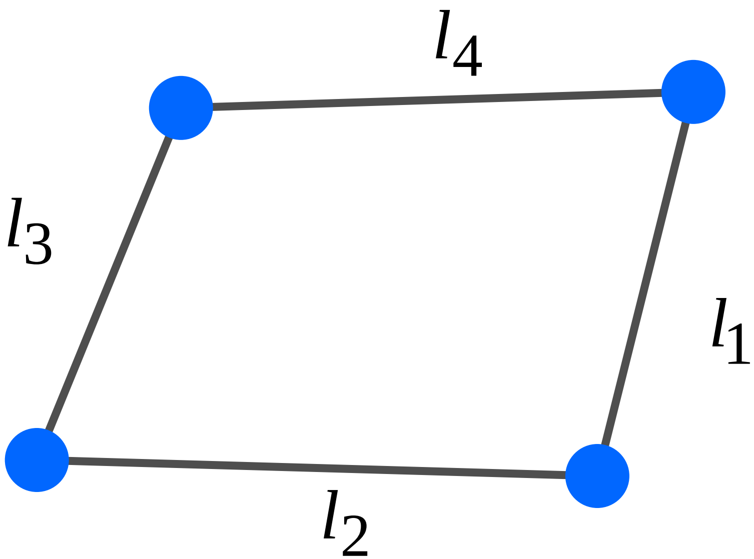
Beam

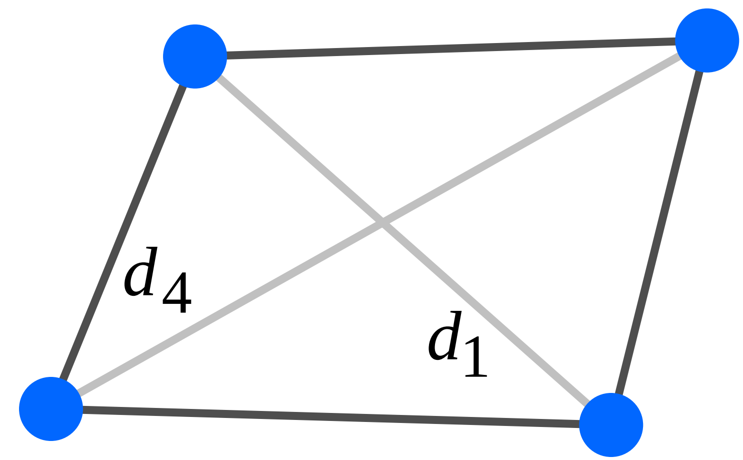
- » Length between two nodes

Excluded elements

- » Discrete beams and springs

Edge or Diagonal Length ?





$$l_c = \min(l_1, l_2, l_3, l_4)$$

$$l_c = \min(d_1, d_2)$$

$$l_c = \max(l_1, l_2, l_3, l_4)$$

$$l_c = \max(d_1, d_2)$$

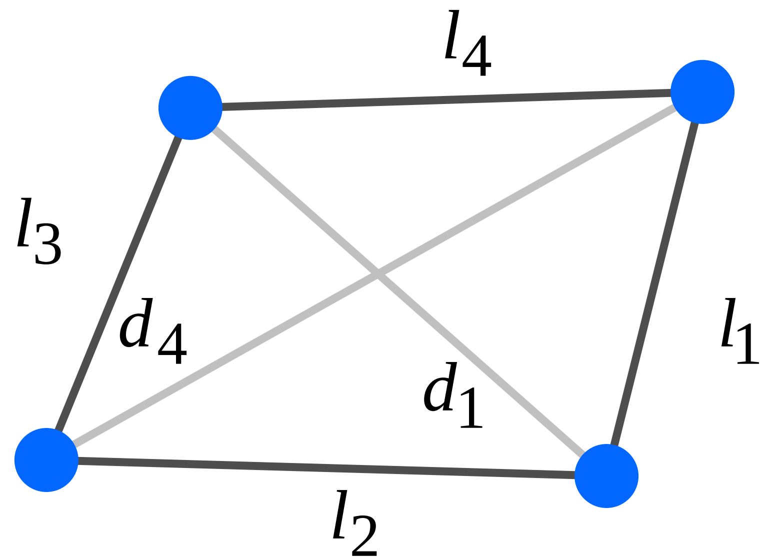
Why Edge and Diagonal Length Fail

Edge or diagonal length based method fails for collapsed or near collapsed elements

$$l_c > 0$$

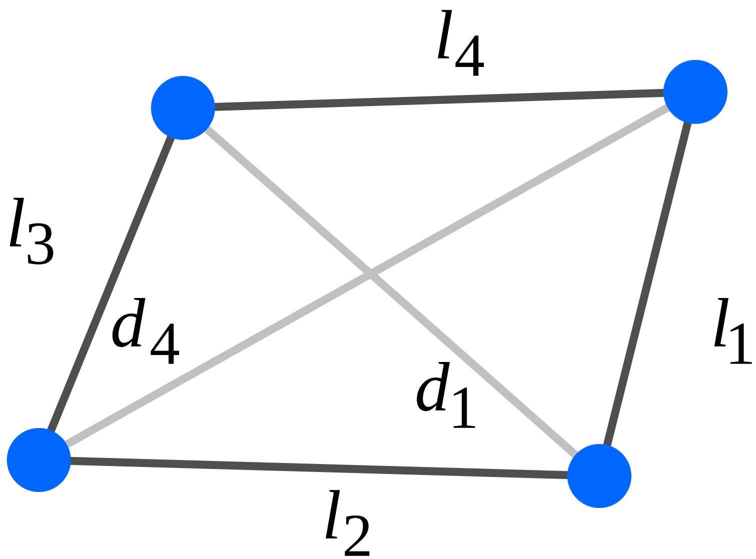
$$A = 0$$

Shell Element Characteristic Length

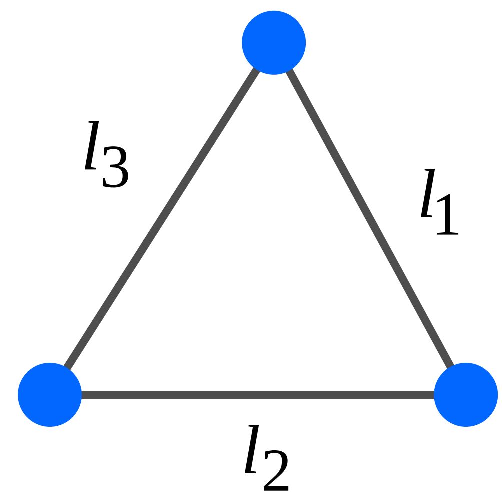


$$l_c = \frac{A}{\max(l_1, l_2, l_3, l_4)}$$

Default Shell Element Characteristic Length



$\beta = 0$



$\beta = 1$

$$l_c = \frac{(1+\beta)A}{\max(l_1, l_2, l_3, (1-\beta)l_4)}$$

Shell Element Characteristic Length - Options

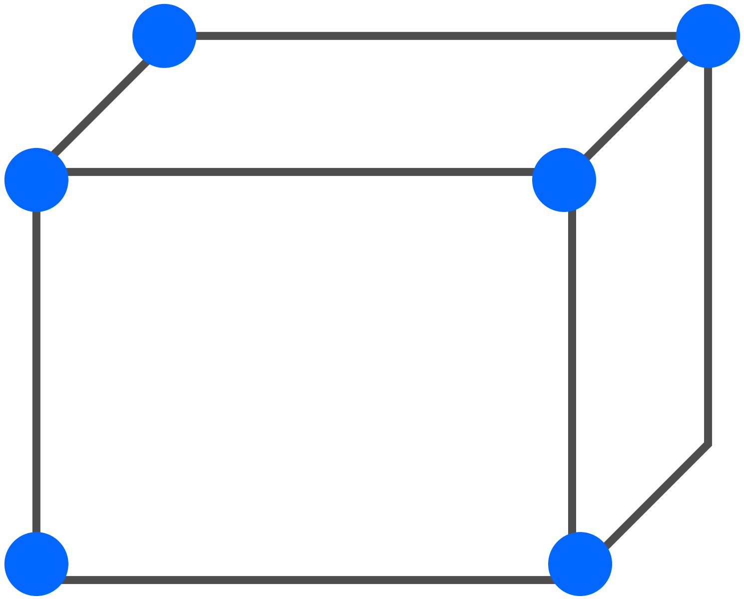
ISDO = 1

$$l_c = \frac{(1+\beta)A}{\max(d_1, d_2)}$$

ISDO = 2

$$l_c = MAX\left[\frac{(1+\beta)A}{\max(l_1, l_2, l_3, (1-\beta)l_4)}, \min(l_1, l_2, l_3, l_4 + \beta 10e20)\right]$$

Solid Element



$$l_c = \frac{Volume}{Area_{\max}}$$

Springs – Timestep

Choosing an incremental timestep is based on the highest natural frequency of the system

$$\Delta t < \frac{2}{\omega_{\max}}$$

$$\omega_{\max} = \frac{k(m_1 + m_2)}{m_1 m_2}$$

$$\Delta t = TSSFAC \; * 2\sqrt{\frac{2 m_1 m_2}{(m_1 + m_2)k}}$$

Wave/Sound Speed, c

$$C_{rod/hughes\_liu\_beam/truss} = \sqrt{\frac{E}{\rho}}$$

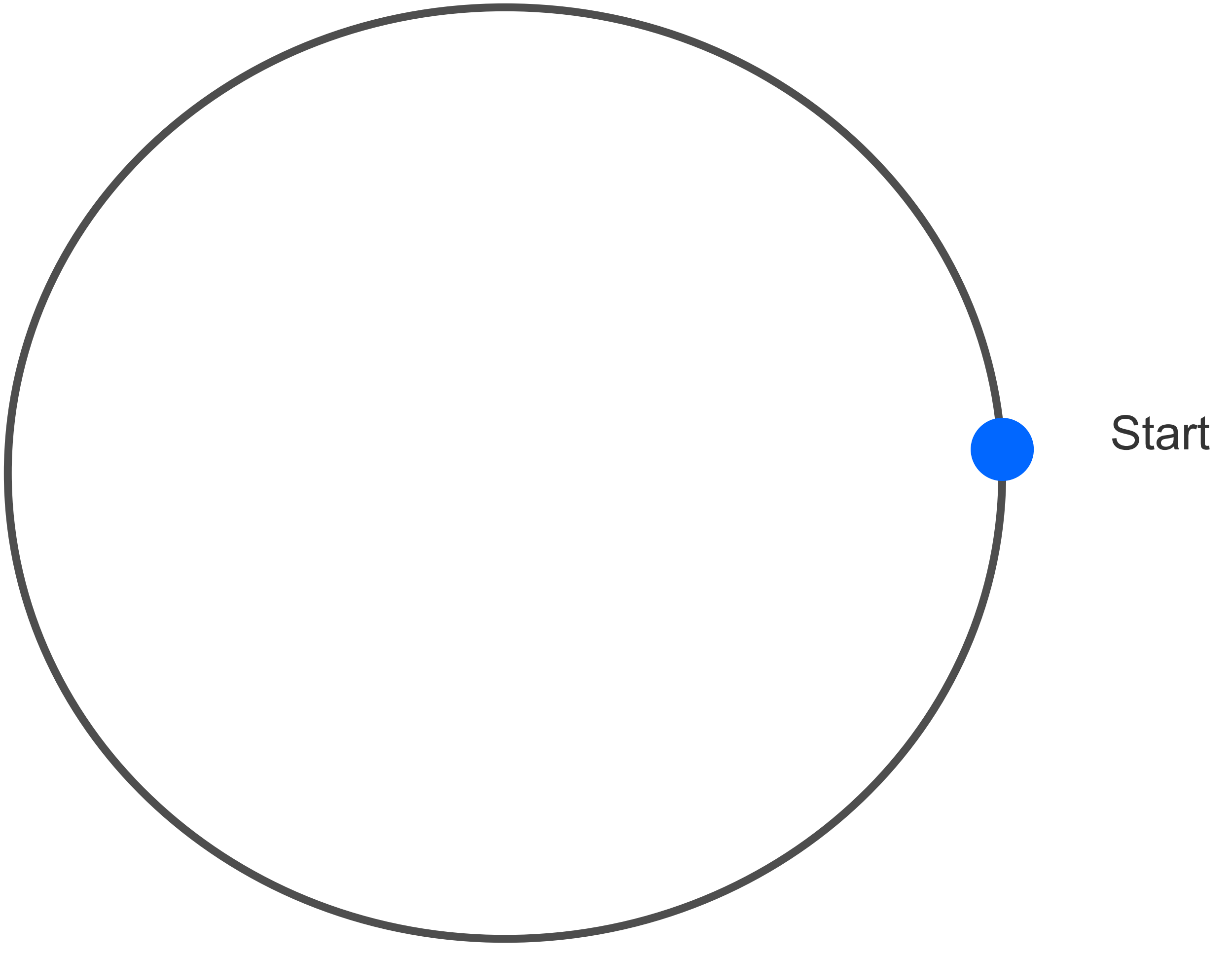
$$C_{shell} = \sqrt{\frac{E}{(1-\upsilon^2)\rho}}$$

$$C_{solid} = \sqrt{\frac{E(1-\upsilon)}{(1+\upsilon)(1-2\upsilon)\rho}}$$

Global Timestep

$$\Delta t = TSSFAC * \min(\Delta t_1, ..., \Delta t_n)$$

Time Integration Loop



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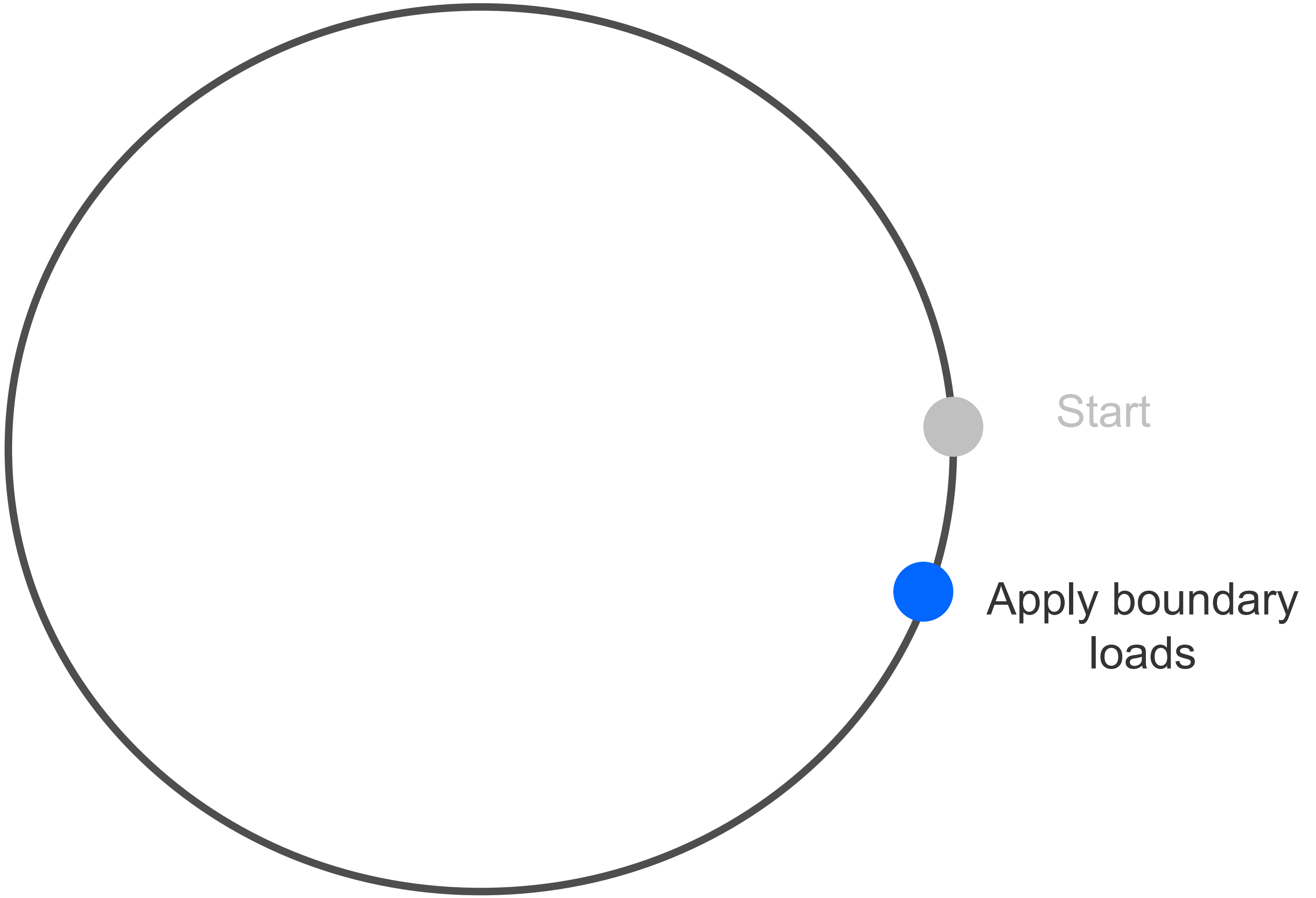

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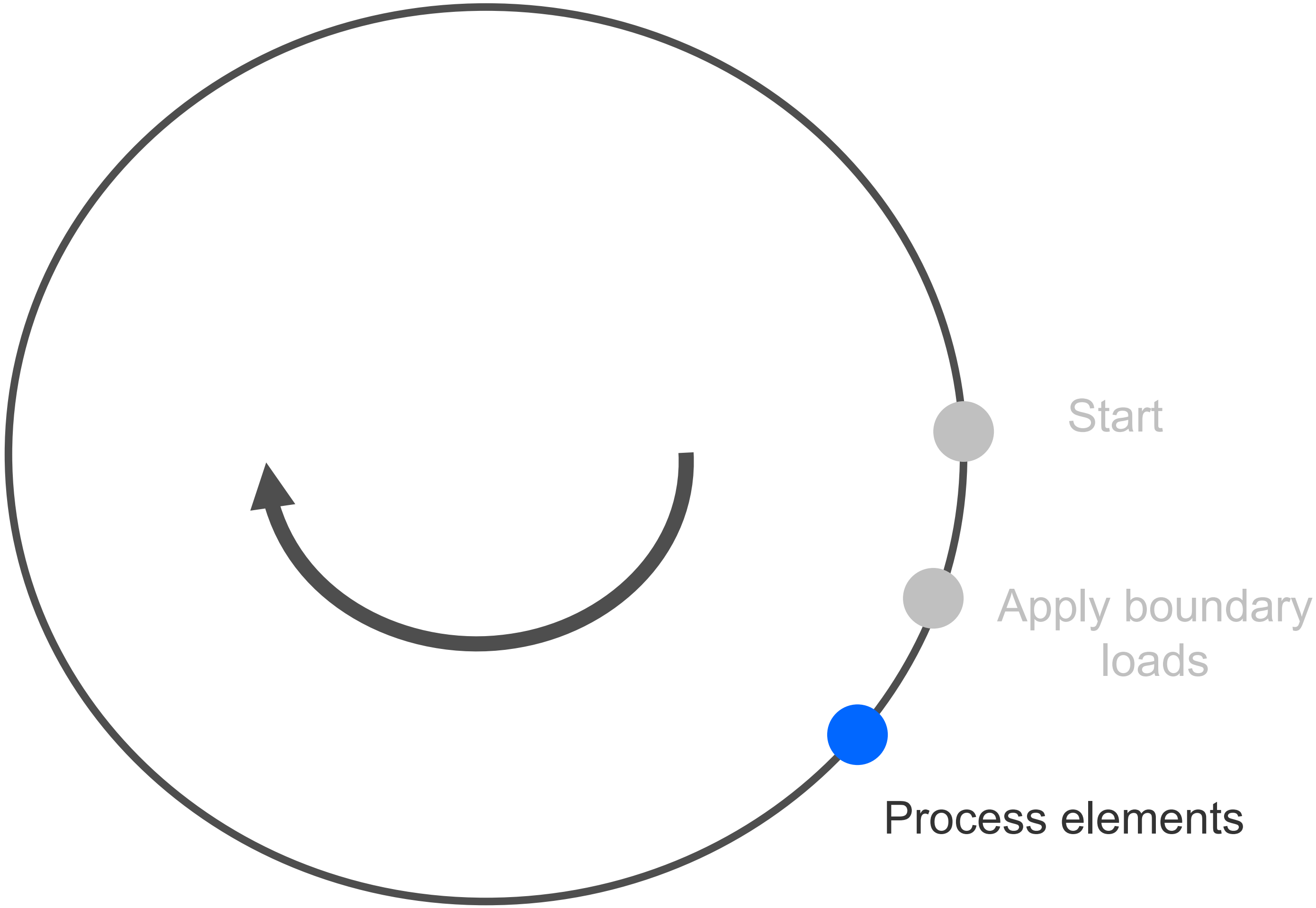

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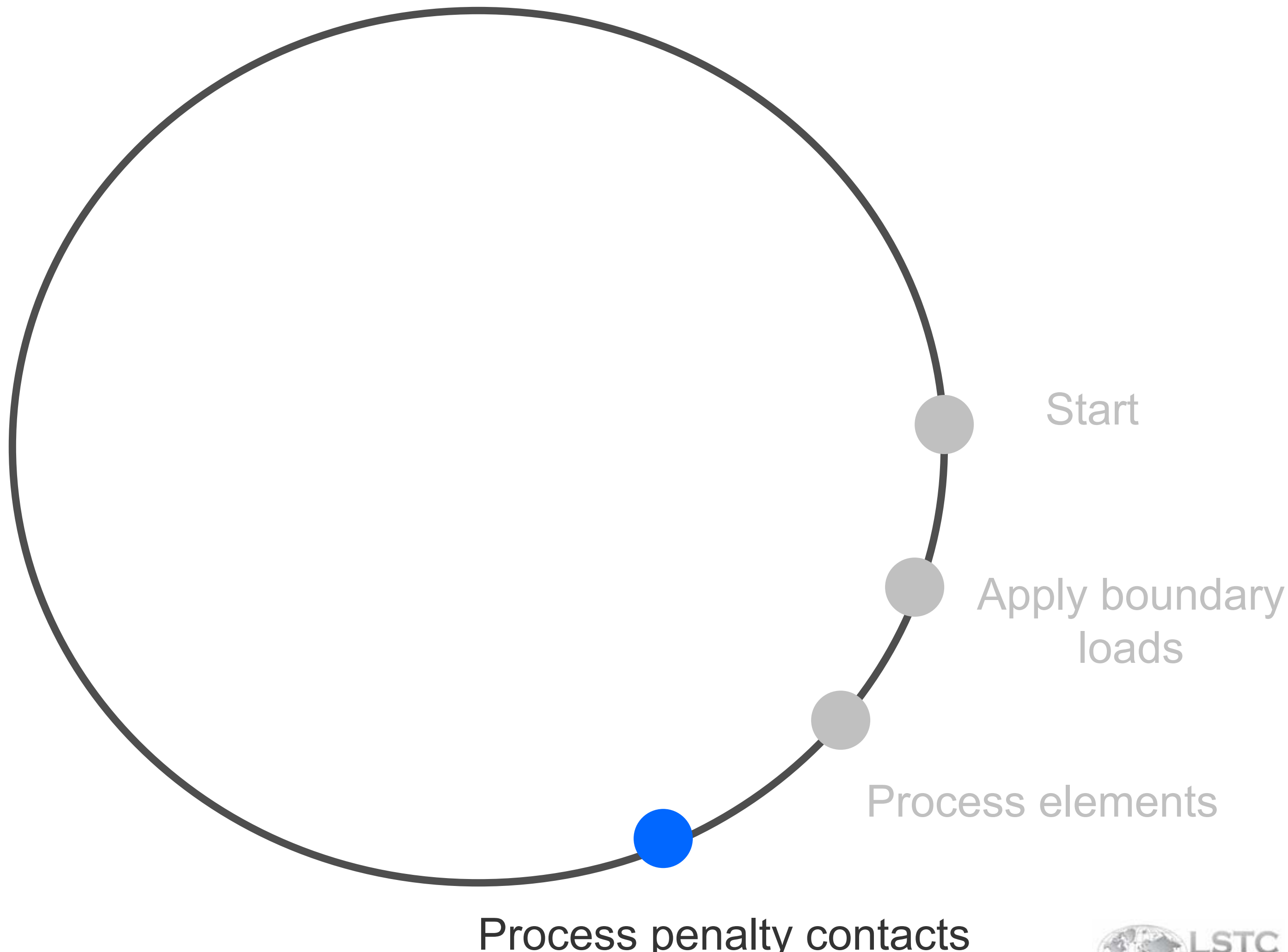

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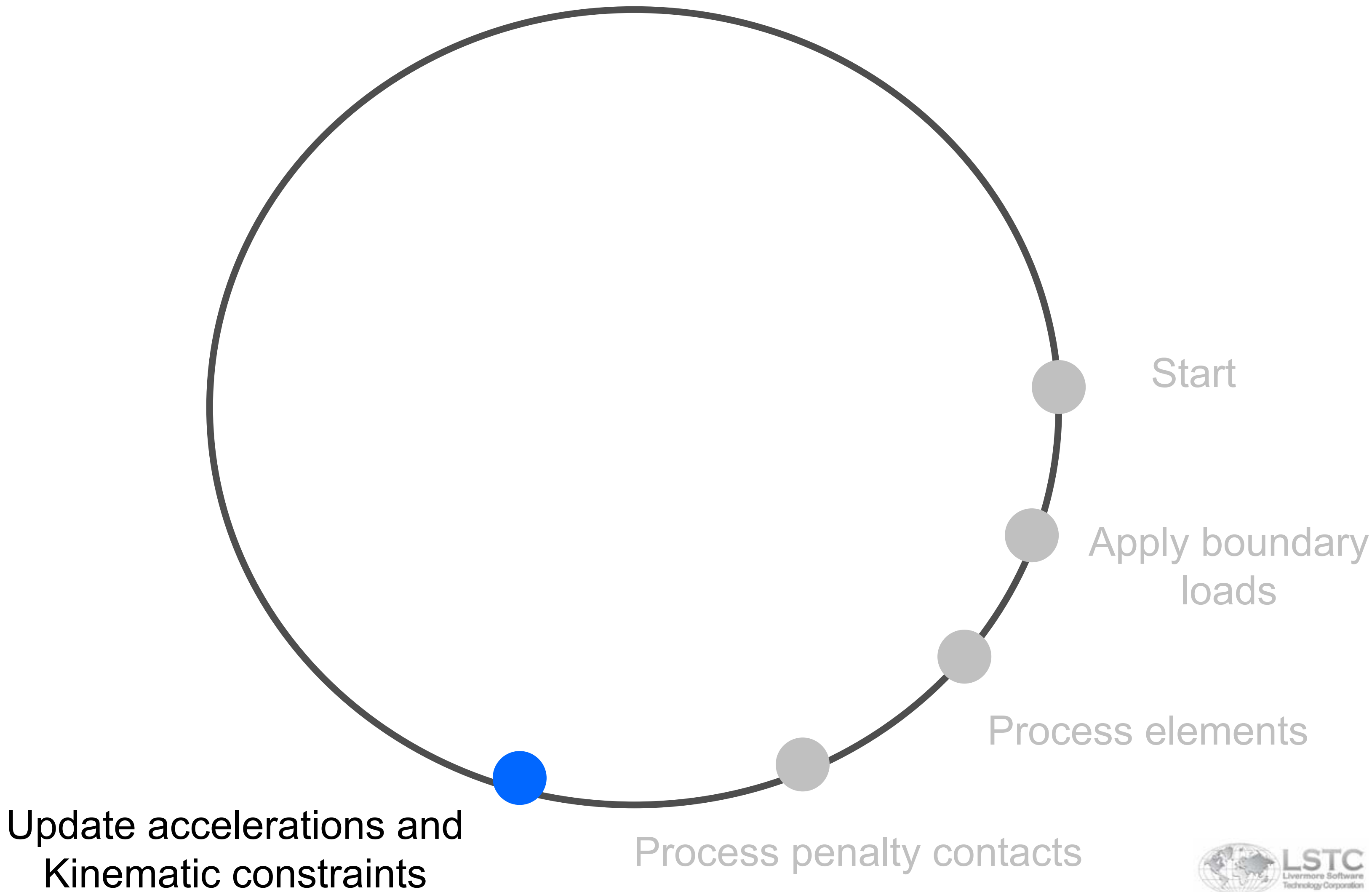

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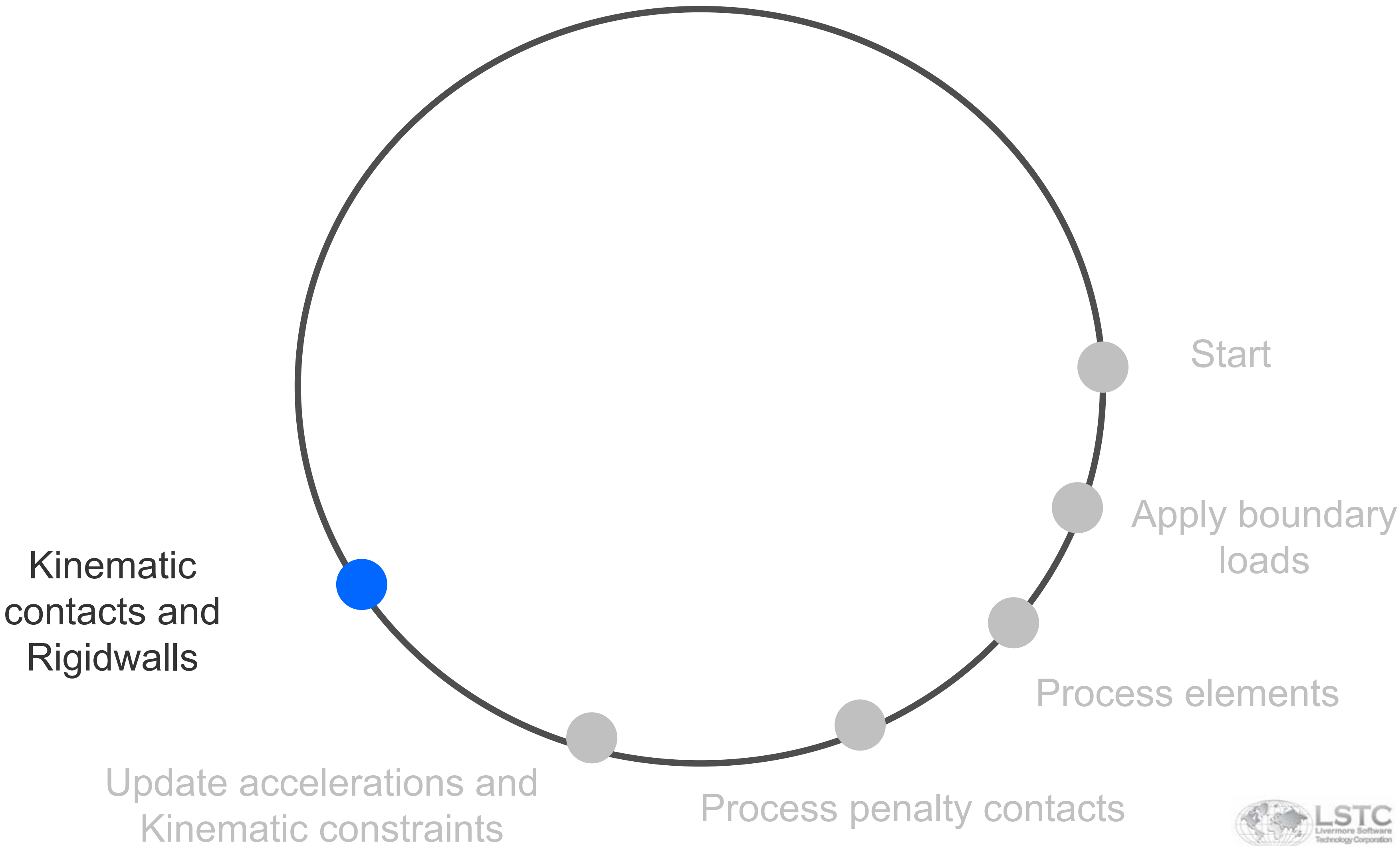

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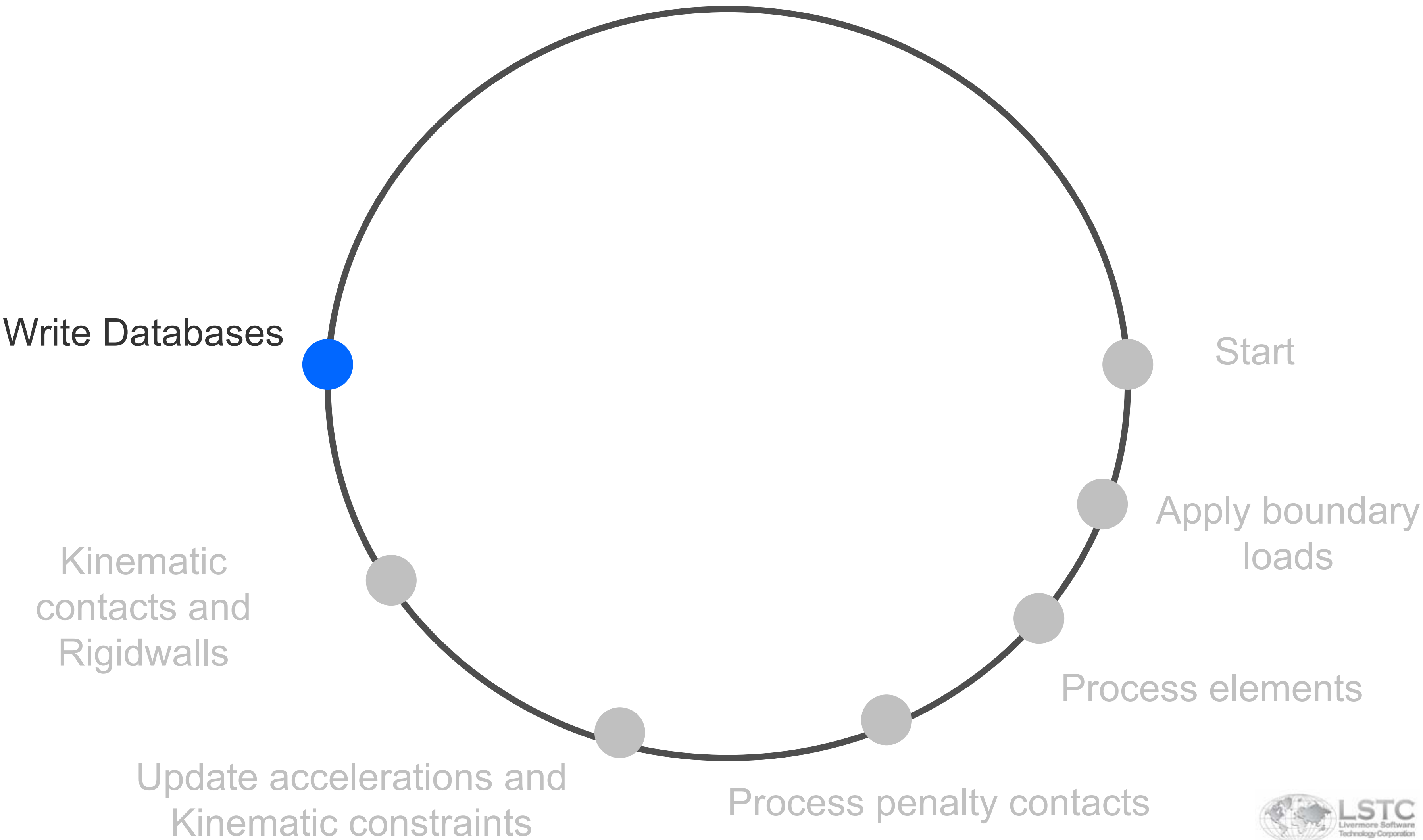

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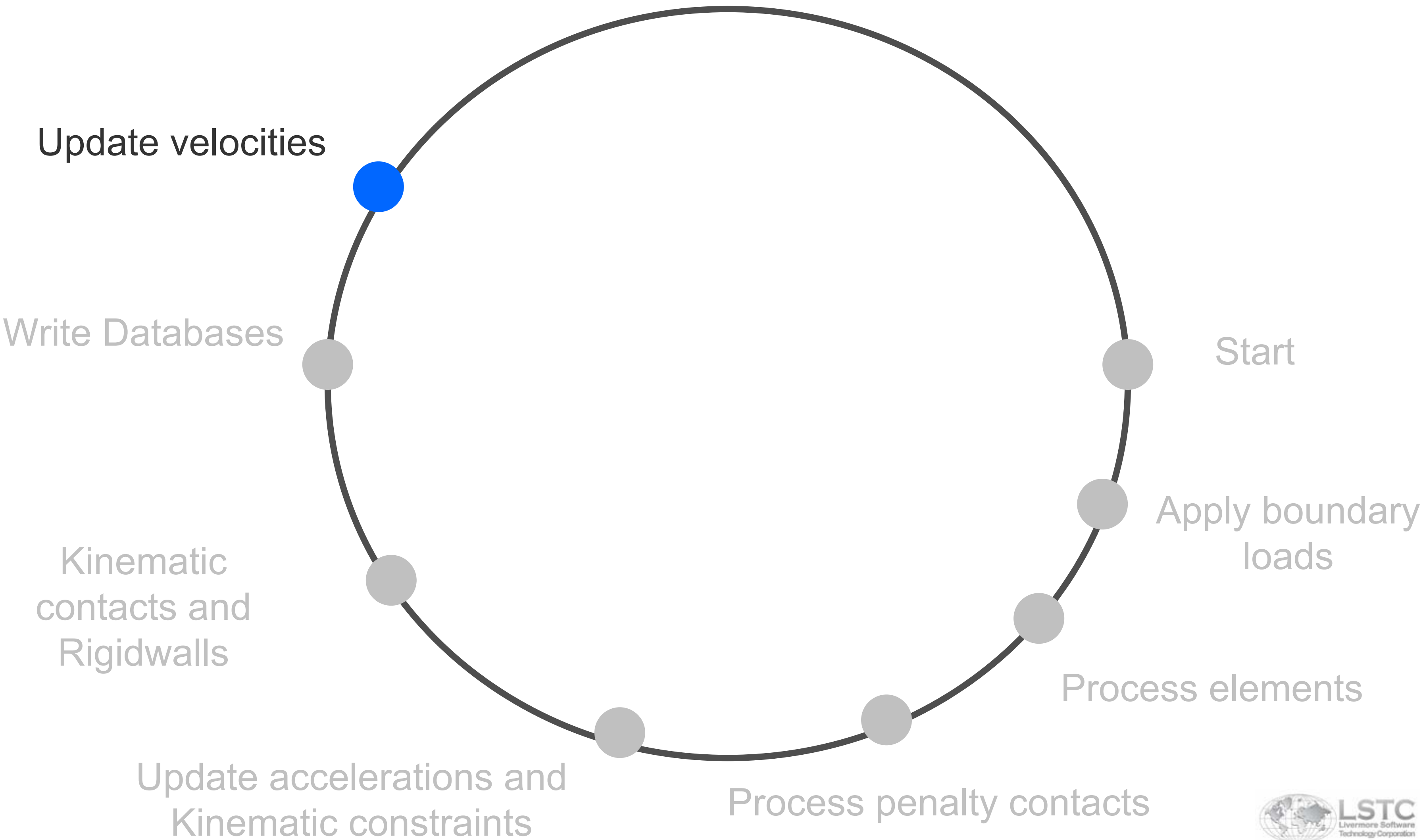

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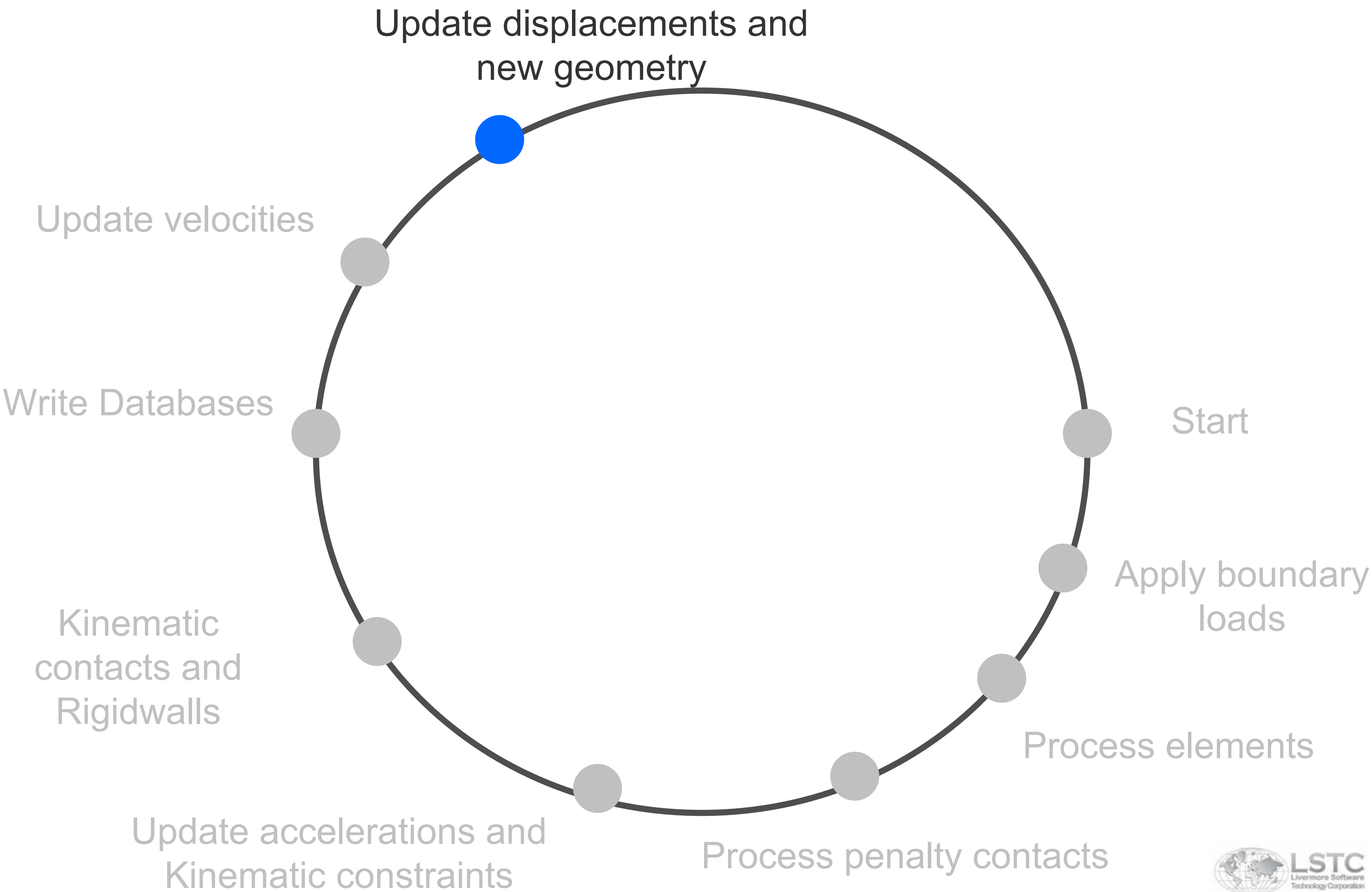

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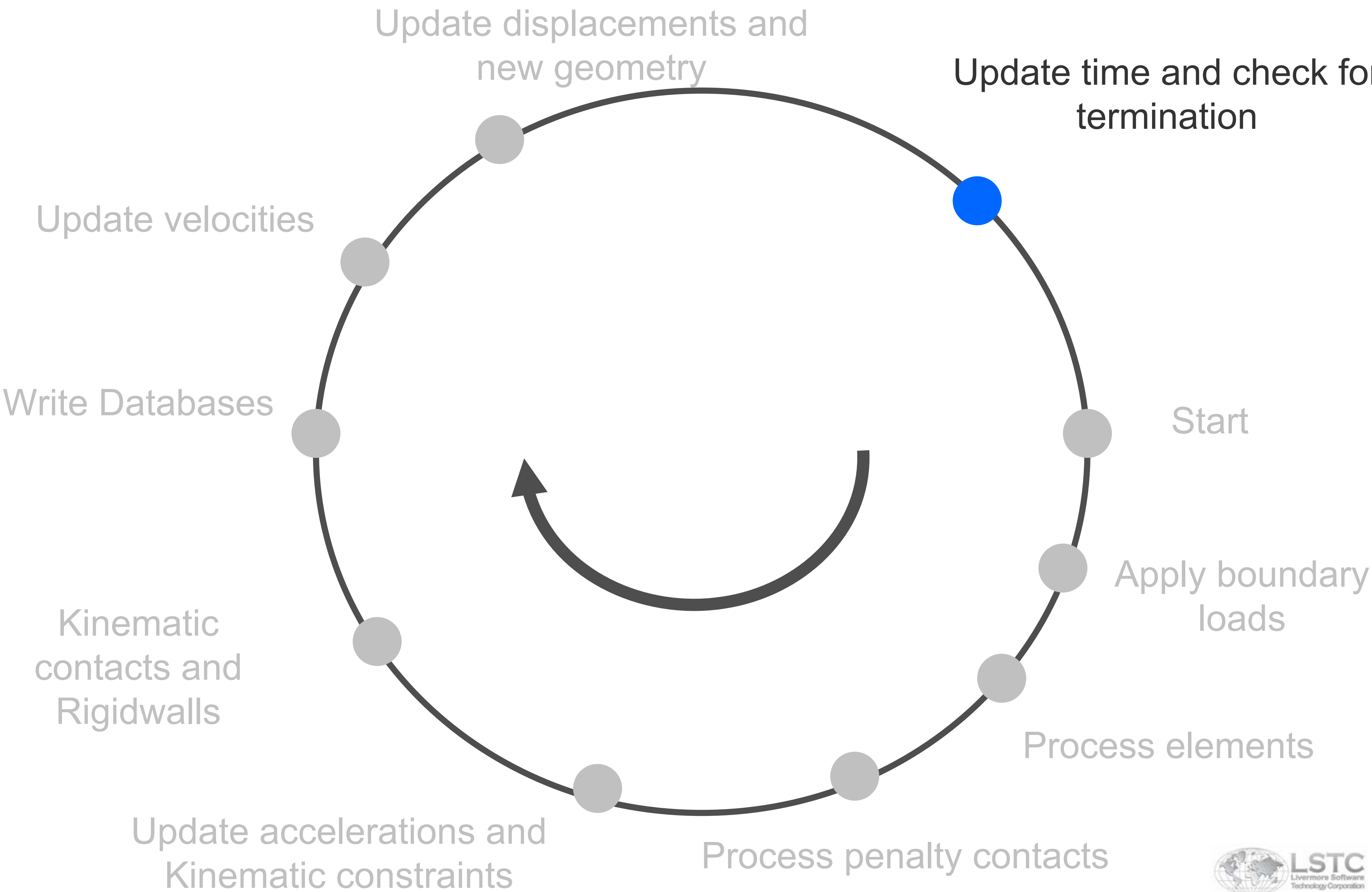

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Increasing CFL based timestep

Stems from the desire to improve job turnaround with negligible effects on accuracy

Two method exists

- Mass Scaling
- Stiffness scaling

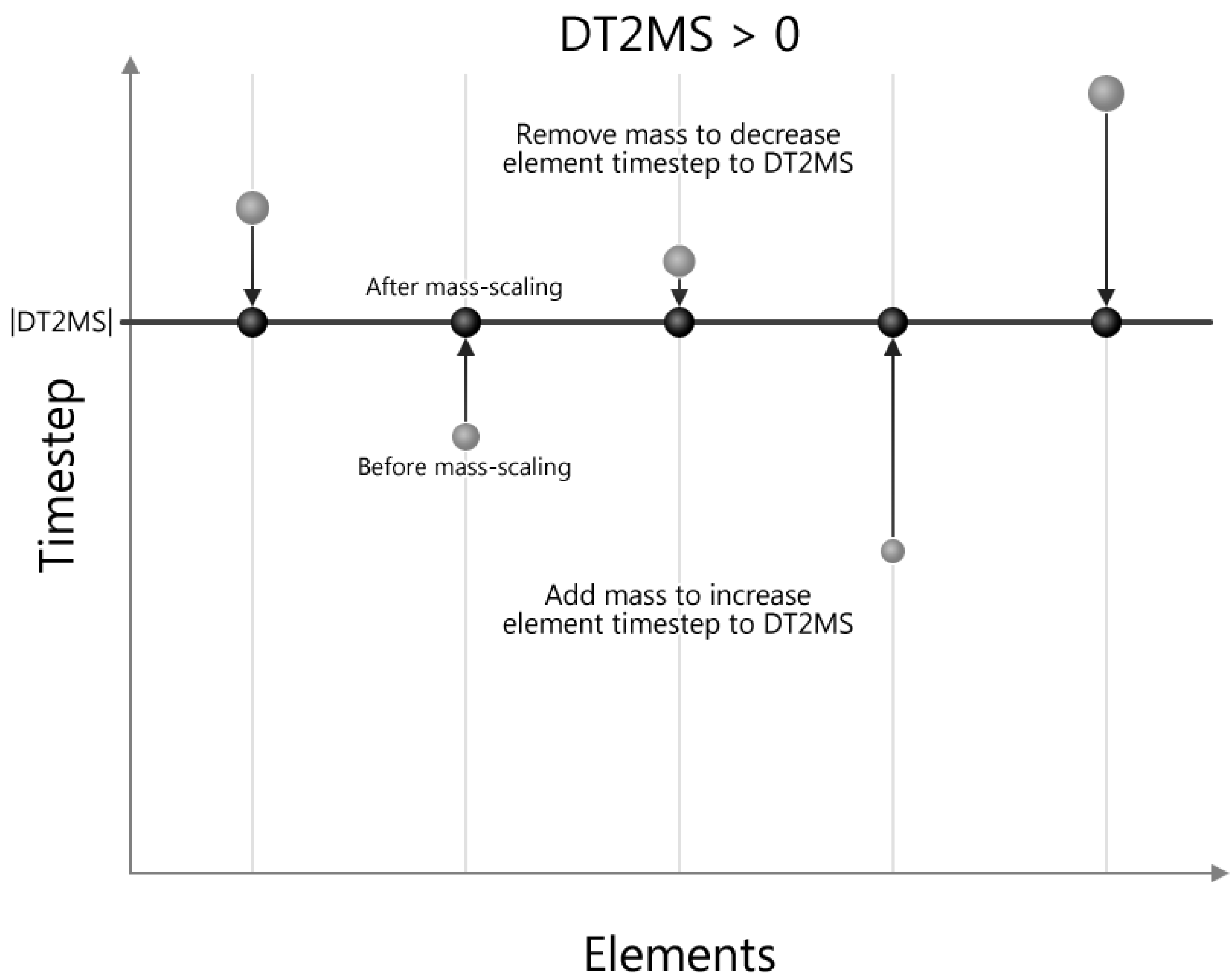
Mass Scaling

- Sound speed is slower in denser materials thereby allows larger timestep

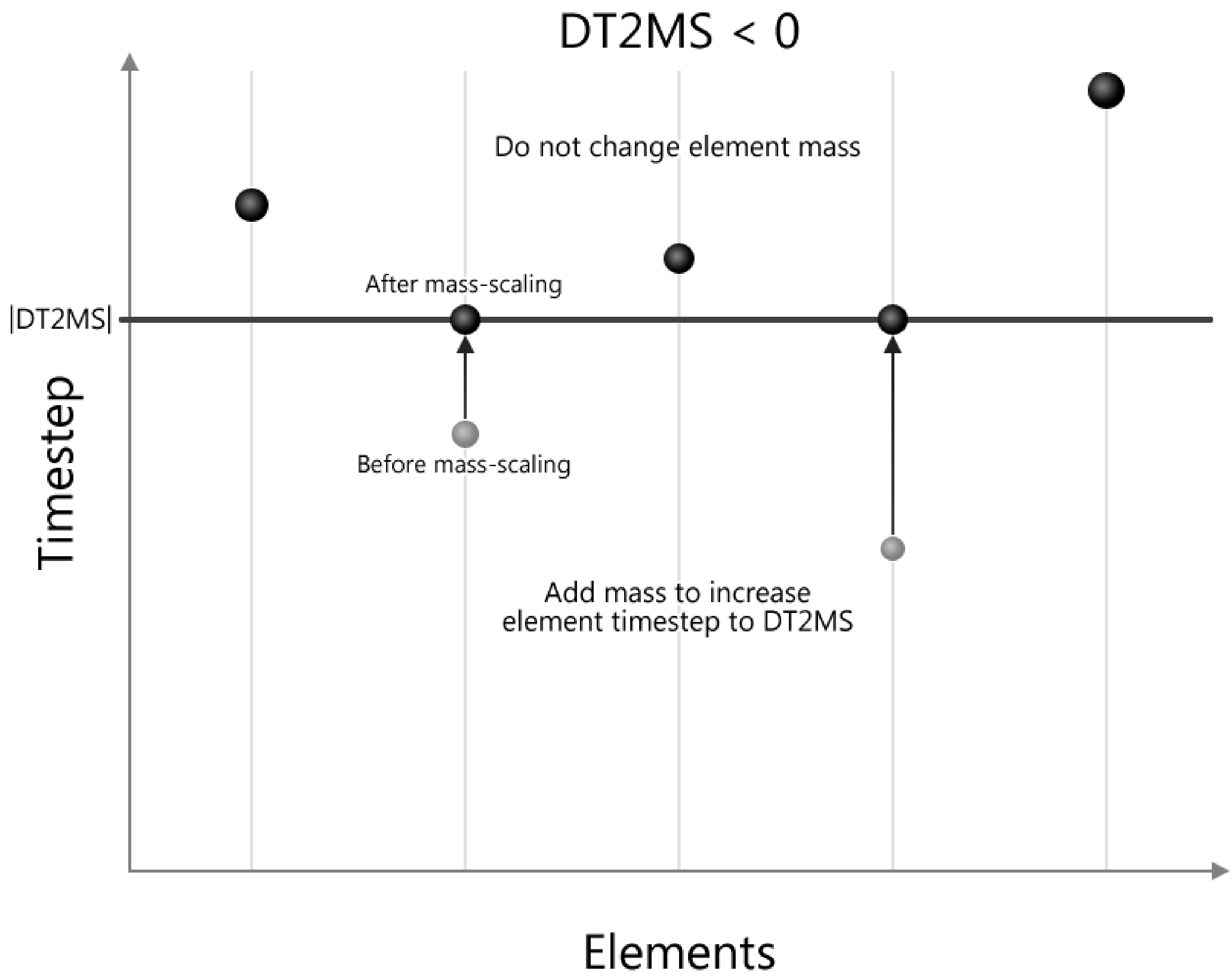
Stiffness

- Sound speed is slower in softer materials thereby allows larger timestep

Mass-Scaling, DT2MS > 0 in *CONTROL_TIMESTEP



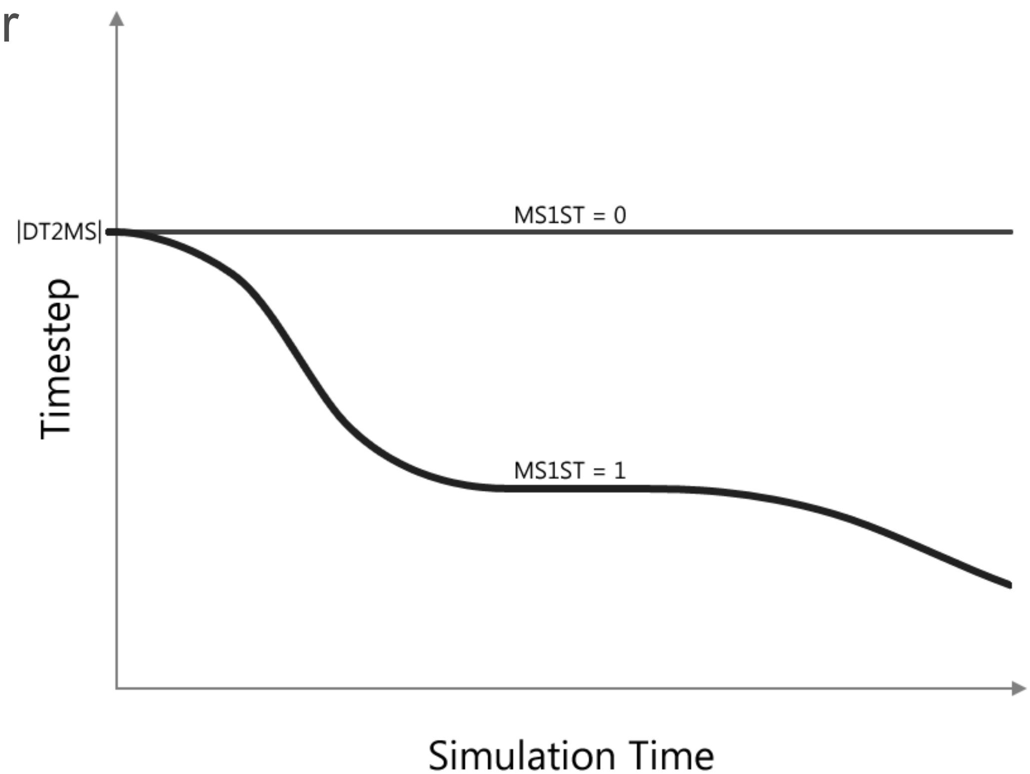
Mass Scaling, DT2MS < 0 in *CONTROL_TIMESTEP



Limiting mass-scaling to first cycle

Mass-scaling is performed at every cycle by default

MS1ST in *CONTROL_TIMESTEP allows to limit the mass-scaling routing to be executed at cycle 1 which allows the timestep to drop thereafter



Selective Mass-Scaling

Available from 971

Allows a larger mass-scaled timestep with negligible reduction in accuracy

Automotive Applications

- Detailed steering wheel
- Any subsystem
- Localized study

Stiffness Scaling

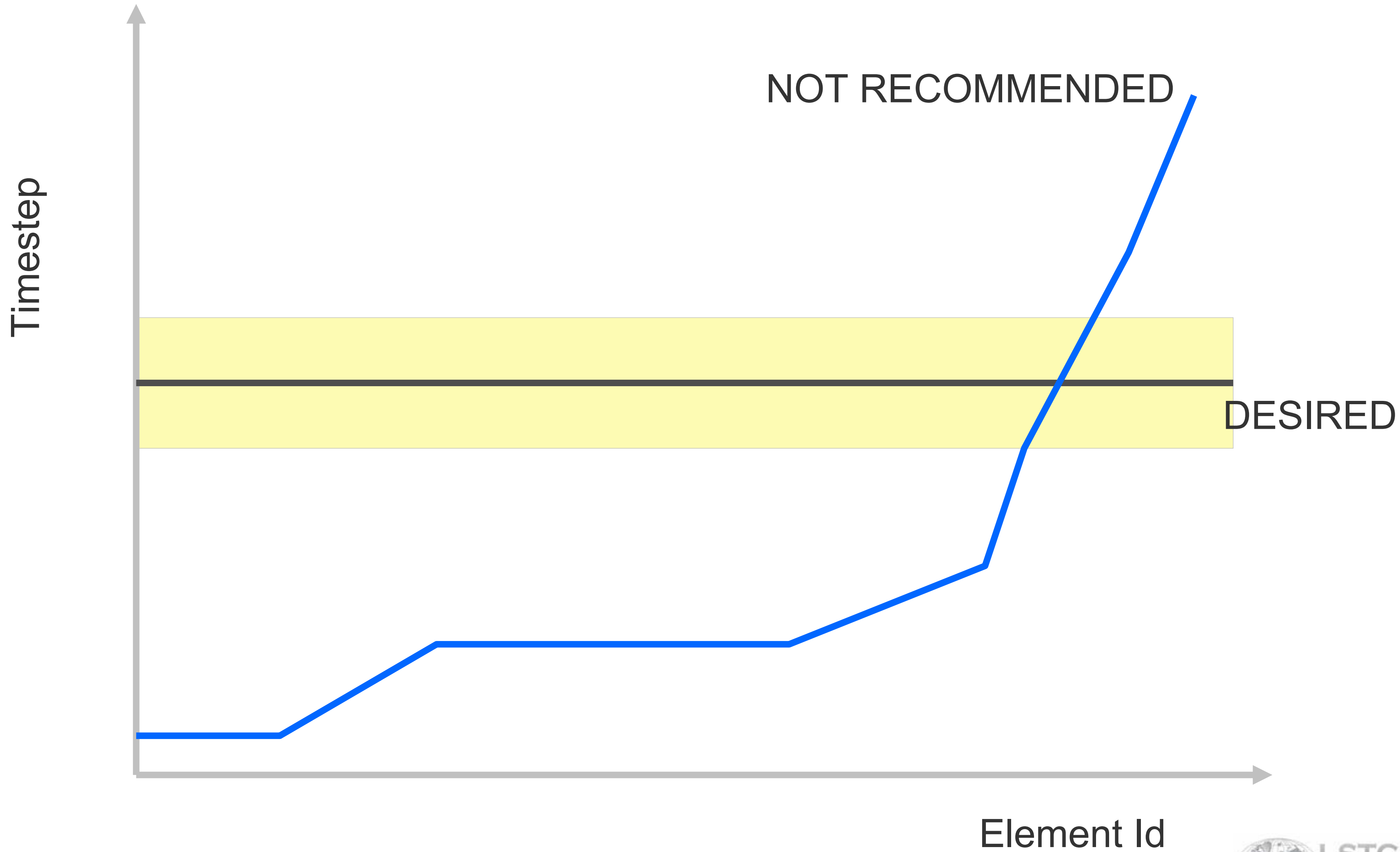
Alternative way of increasing the computed timestep

Alter (reduce) the elastic stiffness, E, to decrease the sound speed thereby increasing the resulting timestep

Options

- Manually by updating the parameter in the *MAT keyword
  - Can be used for ALL element types
- Automatic by specifying the desired timestep, TSMIN, in *CONTROL_TIMESTEP keyword
  - Applies for only shell elements using limited elastic-plastic material laws

100 smallest timesteps in D3HSP



Explicit – Advantages/Disadvantages

+ Ideal for Highly Non-Linear short duration transient events
+ Low Memory requirements
+ Mature Contact treatments
+ Inexpensive Timestep Calculations

- Limited by Courant stability limit
  - Need to ignore geometric details
- Long duration events not feasible

Numerical Solution - Implicit

Unknowns are embedded in a system of linear/non-linear equations

- Stiffness matrix is formed
- Need Efficient Linear/Non-Linear solver

$$\underset{\sim}{x}_1^{\,n+1} = \underset{\sim}{x}^{\,n} + s_0 \Delta \underset{\sim}{u}_0$$

$$\underset{\sim}{K}_{t_j} \Delta \underset{\sim}{u}_i = \underset{\sim}{P}\left(\underset{\sim}{x}_i^{\,n+1}\right)^{n+1} - \underset{\sim}{F}\left(\underset{\sim}{x}_i^{\,n+1}\right) = \underset{\sim}{Q}_i^{\,n+1}$$

Numerical Solution - Implicit

Advantages/Disadvantageous

+ Unconditionally stable for any load/time step
  + Geometric details can be included. A huge benefit for certain automotive structures
+ Ideal for Long and Short Duration events

- High Memory Requirements
- Very expensive Time Step calculations
- Contact Inexperience

Choosing Solution Type

Explicit

- » Short duration
- » High Strain-rate
- » Intertia dominated

Implicit

- » Static
- » Quasi-static
- » Zero to low strain-rates

Combination of both ?

IMFLAG in *CONTROL_GENERAL