Time Integration

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Time Integration

Main objective is to find unknown displacements by numerically integrating the equations of motion which is a second-order linear/nonlinear ODE

For a single degree of freedom with no damping:

\[ m\ddot{x} + kx = f_{ext}(t) \quad \text{Linear} \]
\[ m\ddot{x} + f_{int}(t) = f_{ext}(t) \quad \text{Non-Linear} \]
Numerical Solutions

Direct and Indirect Integration Techniques

Direct
  » No transformations
  » Examples
    ▶ Explicit
    ▶ Implicit

Indirect
  » Transformation
  » Examples
    ▶ Mode Superposition
Numerical Solution - Explicit

Among many explicit methods, the central-difference technique is the most popular and is used in LS-DYNA
Numerical Solution - Explicit

Discretization in Time

\[ F(t) \]
Numerical Solution - Explicit

Current Time

\[ F(t) \]

\[ t_{n-1}, \quad t_n, \quad t_{n+1}, \quad \text{Time} \]
Numerical Solution - Explicit

Current Acceleration

\[ a_n = M^{-1}(P_n - F_n) \]
Numerical Solution - Explicit

Mid-step parameters

\[ a_n = M^{-1}(P_n - F_n) \]
Numerical Solution - Explicit

Mid-Step Velocity

\[ v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + \Delta t a_n \]
Numerical Solution - Explicit

Unknown displacement

\[ d_{n+1} = d_n + \Delta t v_{n+\frac{1}{2}} \]
Choosing an incremental timestep is based on the highest natural frequency of the system

\[ \Delta t < \frac{2}{\omega_{\text{max}}} \]
Explicit – Timestep

Choosing an incremental timestep is based on the highest natural frequency of the system

\[ \Delta t < \frac{2}{\omega_{\text{max}}} \]

\[ \omega_{\text{max}} = \frac{2c}{l} \]
Explicit – Timestep

Choosing an incremental timestep is based on the highest natural frequency of the system

\[ \Delta t < \frac{2}{\omega_{\text{max}}} \]

\[ \omega_{\text{max}} = \frac{2c}{l} \]

\[ \Delta t < \frac{l}{c} \quad \text{Courant-Frederick-Levy (CFL) Criteria} \]
Characteristic Length, $l_c$

Element based

Computed Every Cycle

Beam
   » Length between two nodes

Excluded elements
   » Discrete beams and springs
Edge or Diagonal Length?

\[ l_c = \min(l_1, l_2, l_3, l_4) \]

\[ l_c = \max(l_1, l_2, l_3, l_4) \]

\[ l_c = \min(d_1, d_2) \]

\[ l_c = \max(d_1, d_2) \]
Why Edge and Diagonal Length Fail

Edge or diagonal length based method fails for collapsed or near collapsed elements

\[ l_c > 0 \]

\[ A = 0 \]
Shell Element Characteristic Length

\[ l_c = \frac{A}{\max(l_1, l_2, l_3, l_4)} \]
Default Shell Element Characteristic Length

\[ l_c = \frac{(1 + \beta)A}{\max(l_1, l_2, l_3, (1 - \beta)l_4)} \]
Shell Element Characteristic Length - Options

ISDO = 1

\[ l_c = \frac{(1 + \beta)A}{\max(d_1, d_2)} \]

ISDO = 2

\[ l_c = \max\left[ \frac{(1 + \beta)A}{\max(l_1, l_2, l_3, (1 - \beta) l_4)} , \min(l_1, l_2, l_3, l_4 + \beta 10e20) \right] \]
Solid Element

\[ l_c = \frac{Volume}{\text{Area}_{\text{max}}} \]
Springs – Timestep

Choosing an incremental timestep is based on the highest natural frequency of the system

\[ \Delta t < \frac{2}{\omega_{\text{max}}} \]

\[ \omega_{\text{max}} = \frac{k(m_1 + m_2)}{m_1 m_2} \]

\[ \Delta t = TSSFAC \times 2 \sqrt{\frac{2m_1 m_2}{(m_1 + m_2)k}} \]
Wave/Sound Speed, $c$

\[ C_{\text{rod / hughes_liu_beam/truss}} = \sqrt{\frac{E}{\rho}} \]

\[ C_{\text{shell}} = \sqrt{\frac{E}{(1-\nu^2)\rho}} \]

\[ C_{\text{solid}} = \sqrt{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}} \]
Global Timestep

\[ \Delta t = TSSFAC \times \min(\Delta t_1, \ldots, \Delta t_n) \]
Time Integration Loop
Time Integration Loop

Start

Apply boundary loads
Time Integration Loop

Start → Apply boundary loads → Process elements → Start
Time Integration Loop

- Start
- Apply boundary loads
- Process elements
- Process penalty contacts
Time Integration Loop

Start
Apply boundary loads
Process elements
Process penalty contacts
Update accelerations and Kinematic constraints
Time Integration Loop

- Write Databases
- Start
  - Apply boundary loads
  - Process elements
  - Process penalty contacts
  - Update accelerations and Kinematic constraints
  - Kinematic contacts and Rigidwalls
Time Integration Loop

1. Start
2. Process elements
3. Process penalty contacts
4. Kinematic contacts and Rigidwalls
5. Write Databases
6. Update velocities

Steps:
- Apply boundary loads
- Update accelerations and Kinematic constraints
Time Integration Loop

1. Start
2. Apply boundary loads
3. Process elements
4. Process penalty contacts
5. Update accelerations and Kinematic constraints
6. Kinematic contacts and Rigidwalls
7. Write Databases
8. Update velocities
9. Update displacements and new geometry
Time Integration Loop

1. Start
2. Process elements
3. Process penalty contacts
4. Update accelerations and Kinematic constraints
5. Kinematic contacts and Rigidwalls
6. Write Databases
7. Update velocities
8. Update displacements and new geometry
9. Update time and check for termination
Increasing CFL based timestep

Stems from the desire to improve job turnaround with negligible effects on accuracy

Two methods exist
  » Mass Scaling
  » Stiffness scaling

Mass Scaling
  » Sound speed is slower in denser materials thereby allows larger timestep

Stiffness
  » Sound speed is slower in softer materials thereby allows larger timestep
Mass-Scaling, DT2MS > 0 in *CONTROL_TIMESTEP

- DT2MS > 0
  - Remove mass to decrease element timestep to DT2MS
  - Add mass to increase element timestep to DT2MS

Before mass-scaling

After mass-scaling

Timestep

Elements
Mass Scaling, DT2MS < 0 in *CONTROL_TIMESTEP

- DT2MS < 0
  - Do not change element mass
  - Add mass to increase element timestep to DT2MS

Diagram:
- Timestep
- |DT2MS|
- Elements
- Before mass-scaling
- After mass-scaling
Limiting mass-scaling to first cycle

Mass-scaling is performed at every cycle by default.

MS1ST in *CONTROL_TIMESSTEP allows to limit the mass-scaling routing to be executed at cycle 1 which allows the timestep to drop thereafter.
Selective Mass-Scaling

Available from 971

Allows a larger mass-scaled timestep with negligible reduction in accuracy

Automotive Applications
  » Detailed steering wheel
  » Any subsystem
  » Localized study
Stiffness Scaling

Alternative way of increasing the computed timestep

Alter (reduce) the elastic stiffness, $E$, to decrease the sound speed thereby increasing the resulting timestep

Options

» Manually by updating the parameter in the *MAT keyword
  ▶ Can be used for ALL element types

» Automatic by specifying the desired timestep, TSMIN, in *CONTROL_TIMESTEP keyword
  ▶ Applies for only shell elements using limited elastic-plastic material laws
100 smallest timesteps in D3HSP
Explicit – Advantages/Disadvantages

+ Ideal for Highly Non-Linear short duration transient events
+ Low Memory requirements
+ Mature Contact treatments
+ Inexpensive Timestep Calculations

- Limited by Courant stability limit
  Need to ignore geometric details
- Long duration events not feasible
Numerical Solution - Implicit

Unknowns are embedded in a system of linear/non-linear equations

- Stiffness matrix is formed
- Need Efficient Linear/Non-Linear solver

\[
x_{n+1}^{\sim 1} = x_n^{\sim 0} + s_0 \Delta u^{\sim 0}
\]

\[
K \Delta u^{\sim t_i} = P \left( x_n^{n+1} \right)^{n+1} - F(x^{n+1}) = Q^{n+1}
\]
Numerical Solution - Implicit

Advantages/Disadvantageous

+ Unconditionally stable for any load/time step
  + Geometric details can be included. A huge benefit for certain automotive structures
  + Ideal for Long and Short Duration events

- High Memory Requirements
- Very expensive Time Step calculations
- Contact Inexperienced
Choosing Solution Type

Explicit
  » Short duration
  » High Strain-rate
  » Intertia dominated

Implicit
  » Static
  » Quasi-static
  » Zero to low strain-rates

Combination of both?
IMFLAG in *CONTROL_GENERAL